Critical-Point Nuclei

- Introduction to critical-point descriptions of nuclear shape transitions.

- Searching for X(5) behavior (the transition from spherical vibrator to axially deformed rotor).

- Band-mixing as an alternative picture.

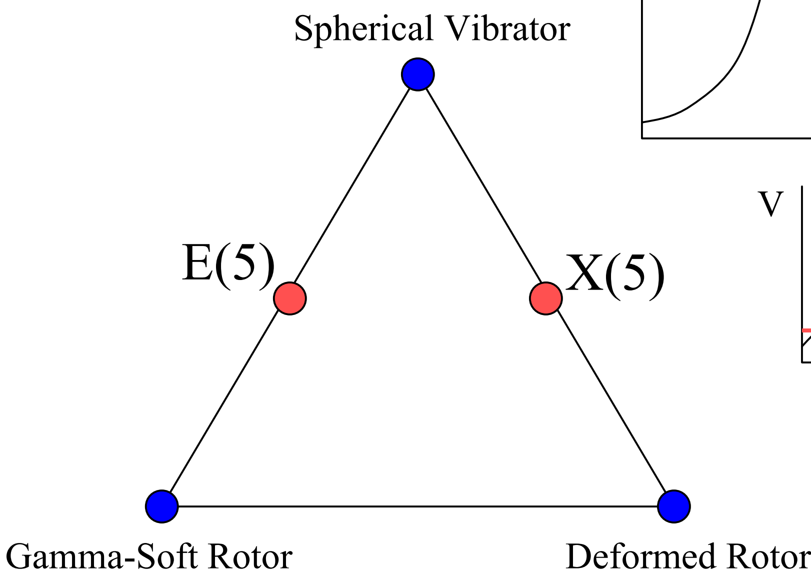
- Searching for E(5) behavior (the transition from spherical vibrator to gamma-soft rotor).

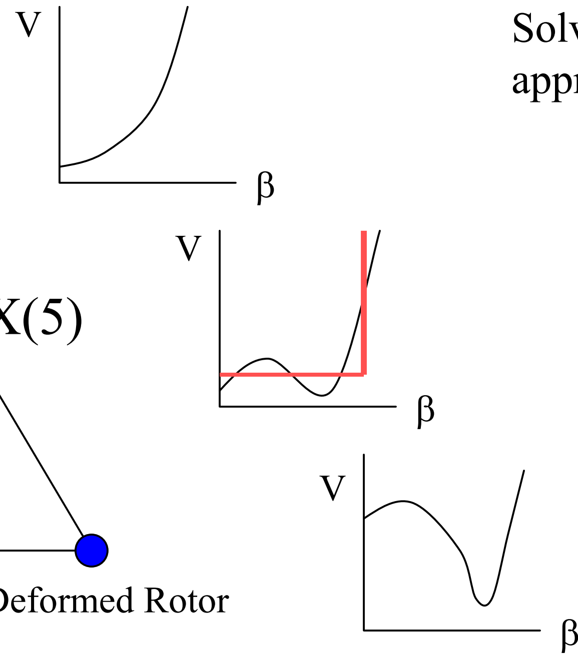
- Modifications to critical-point descriptions.

- The pairing phase: from pairing vibrations to pairing rotations.

- Summary.

Critical-Point Nuclei





Solve Bohr Collective Hamiltonian using approximations of "true" potential

$$H=T+V(\beta,\gamma)$$

X(5): $V(\beta,\gamma)=V(\beta)+V(\gamma)$
$V(\beta)$=infinite square well
$V(\gamma)$=harmonic oscillator

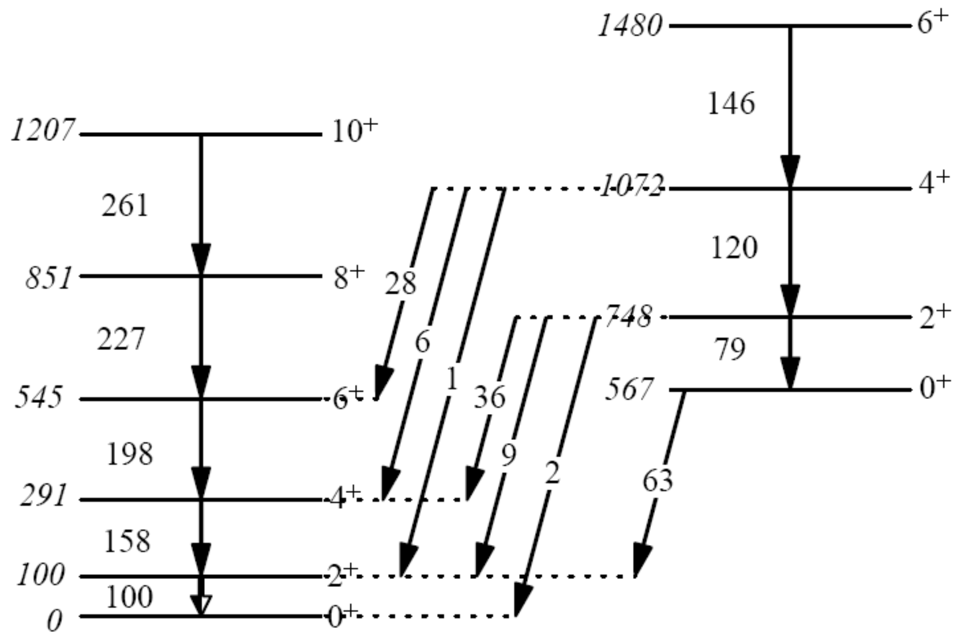
E(5): $V(\beta,\gamma)=V(\beta)$
$V(\beta)$=infinite square well

F.Iachello, PRL 87 (2001) 052502
PRL 85 (2000) 3580

- Analytic solutions (related to Bessel functions)
- Few free parameters (usually only two to fix scales)
- Unphysical (infinite, square wells)
- Ignores microscopic properties (such as pairing)

Can such simple solutions apply to real nuclei?

X(5): Spherical Vibrator to Axially Deformed Rotor Characteristics



a) Energies of the yrast states [$E(4_1^+)/E(2_1^+) \approx 2.91$]

b) B(E2)'s of the yrast transitions

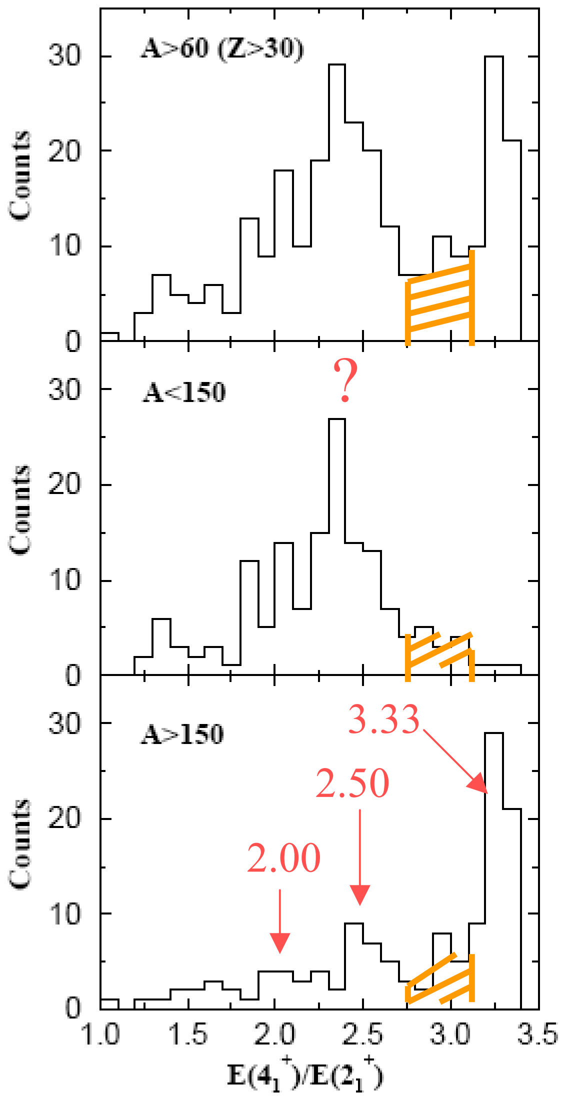
c) 0_2^+ energy [$E(0_2^+) \approx 5.67 \times E(2_1^+)$]

d) Larger energy spacings in excited bands

e) Lower B(E2) values in excited band

f) Pattern of inter-sequence B(E2)'s

Searching for X(5) Behavior



All nuclei with $Z \geq 30$, $A \geq 60$

$E(4_1^+)/E(2_1^+)=3.33$ for axial rotor

$E(4_1^+)/E(2_1^+)=2.50$ for γ-soft rotor

$E(4_1^+)/E(2_1^+)=2.00$ for spherical vibrator

X(5): $2.71 \leq E(4_1^+)/E(2_1^+) \leq 3.11$

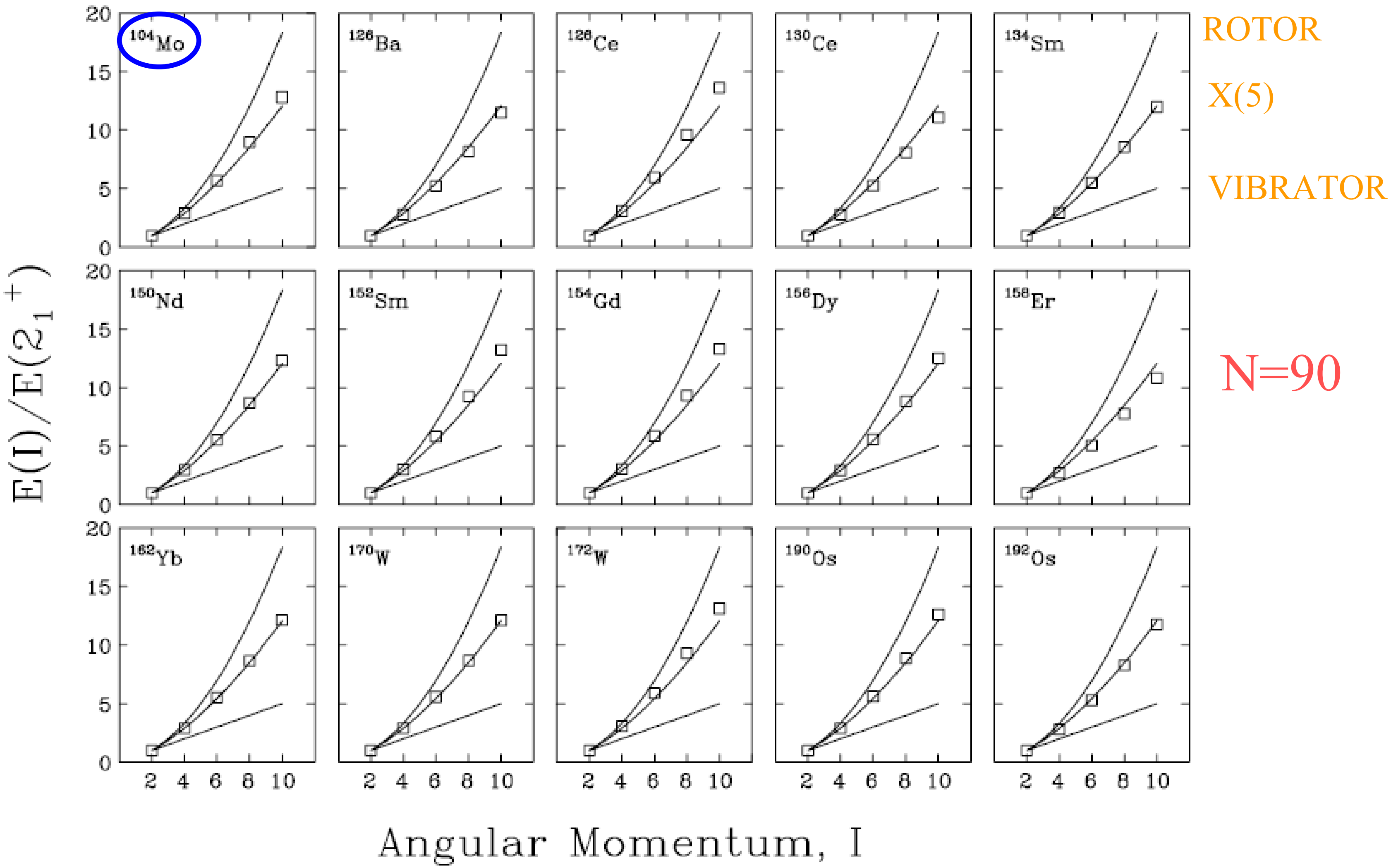
35 Candidates

Why is there such a pronounced peak near $E(4_1^+)/E(2_1^+)=2.3$ for A<150?

R.M.Clark et al., Phys. Rev. C 68 (2003) 037301

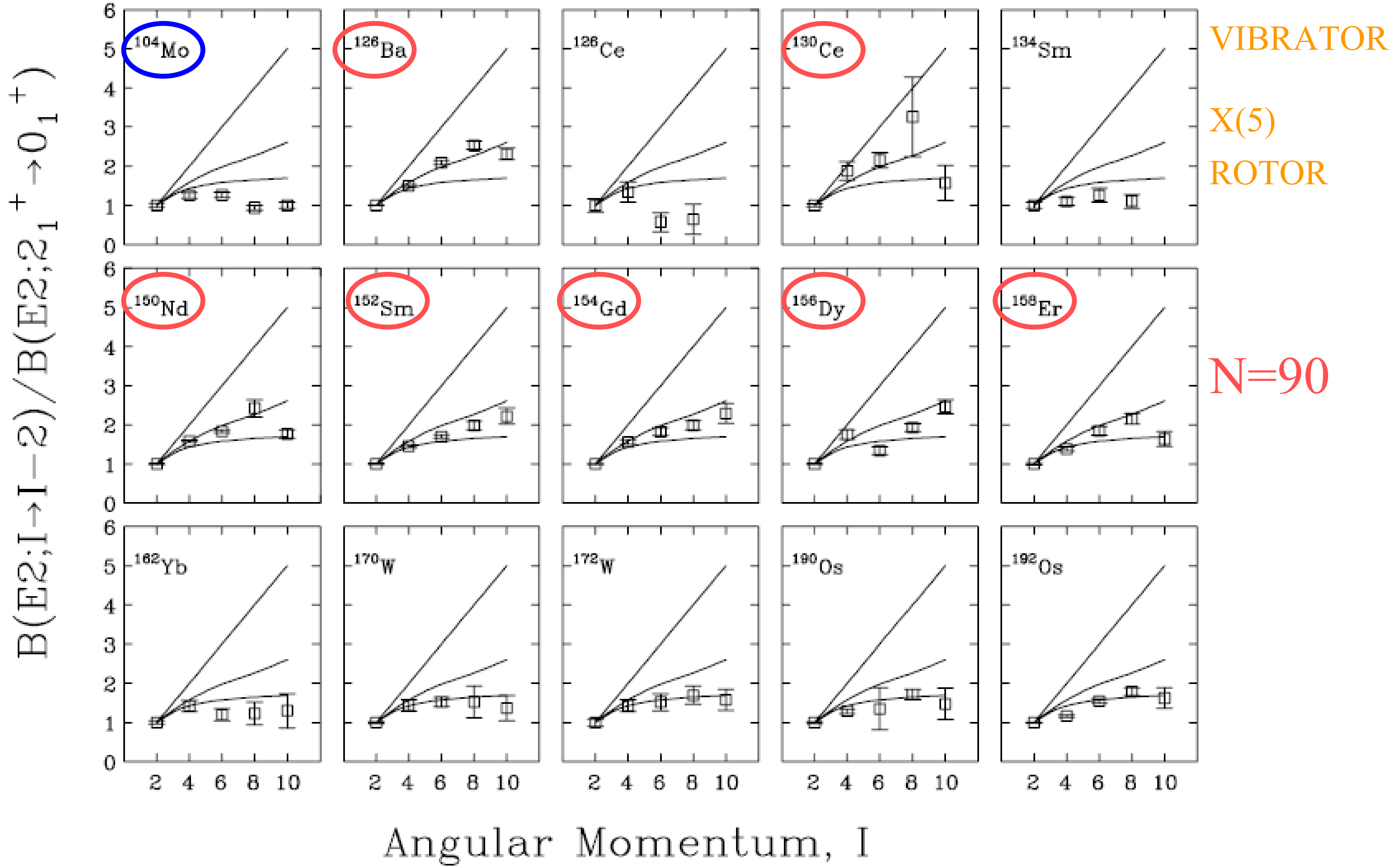
Energy Ratios

Fifteen X(5) candidate nuclei have accurate lifetimes measured



B(E2) Ratios

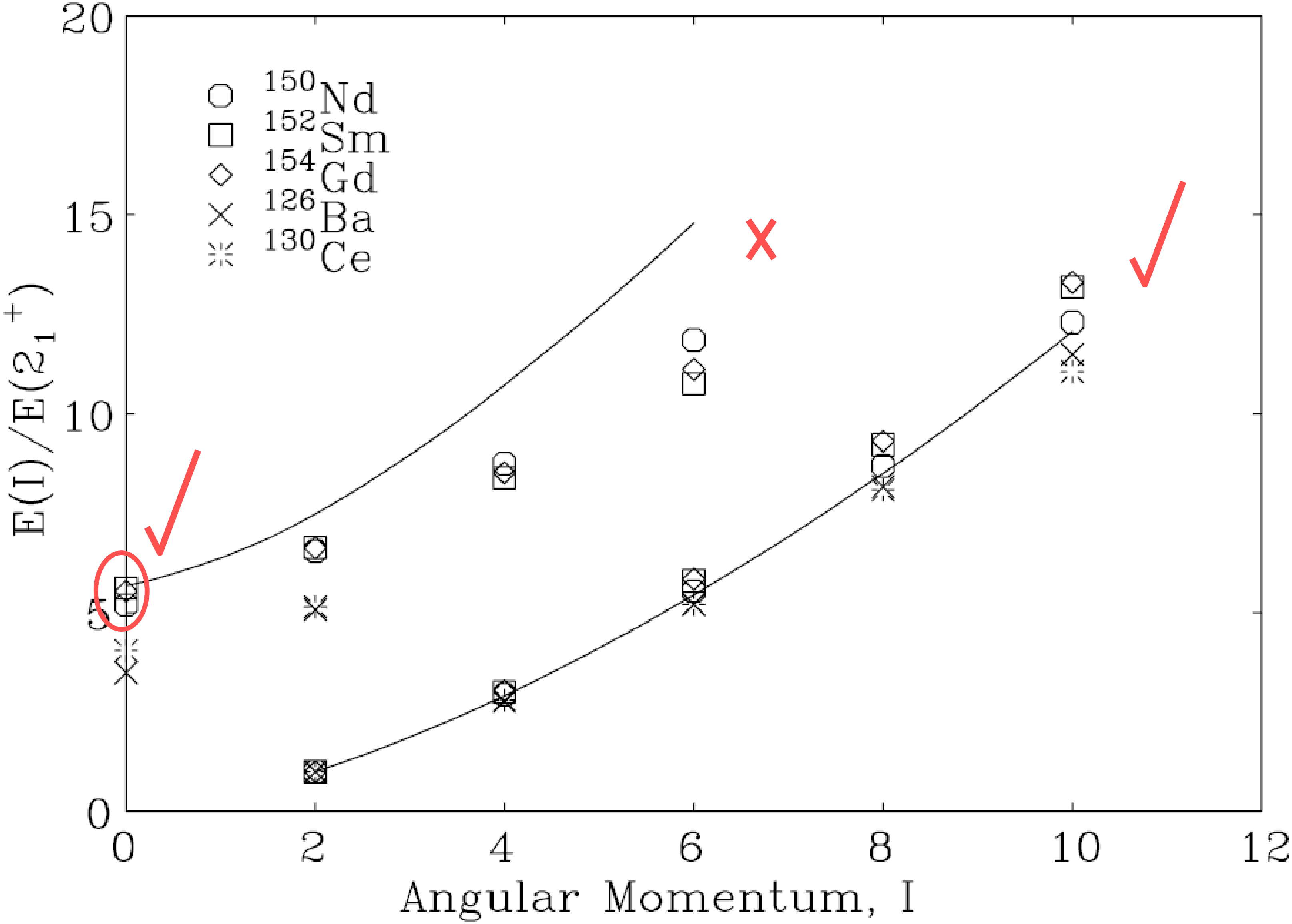
Sufficient to exclude many X(5) candidates



^{150}Nd: R.Kruecken et al., Phys. Rev. Lett. 88 (2002) 232501

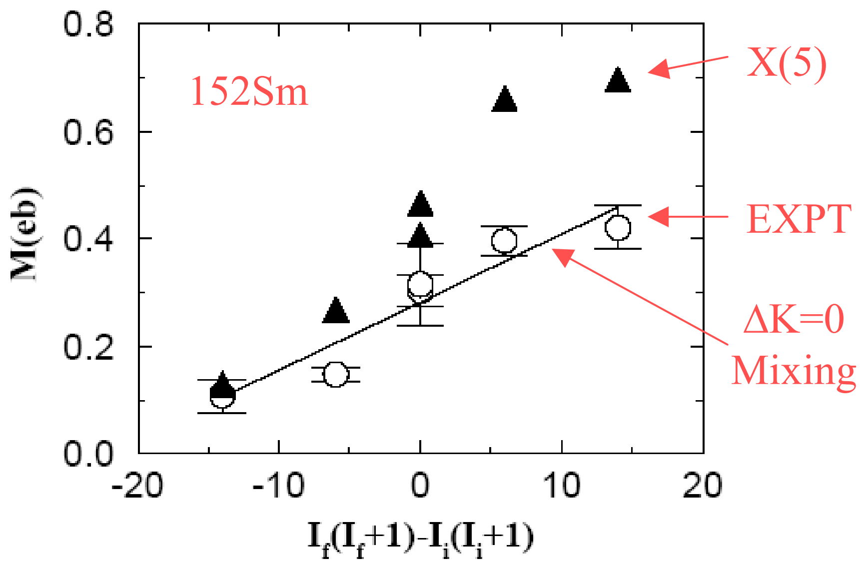
^{152}Sm: R.F.Casten and N.V.Zamfir, Phys. Rev. Lett. 87 (2001) 052502

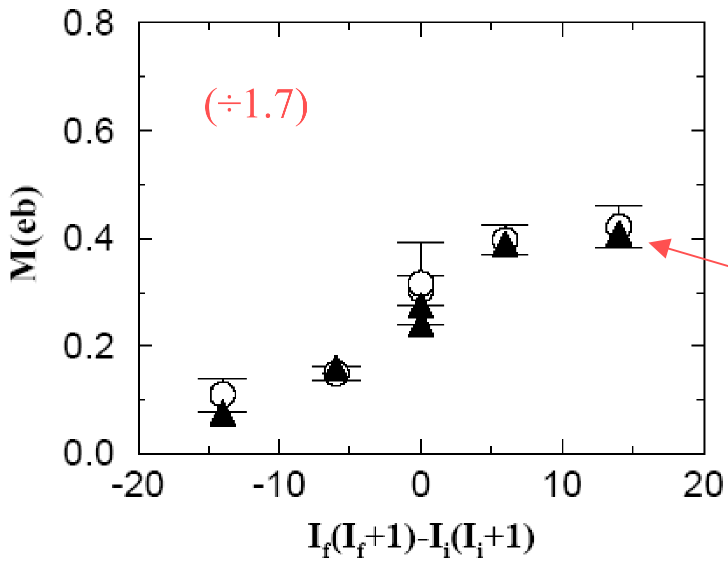
Energies of Excited States



Inter-Sequence Matrix Elements

X(5) does not reproduce properties involving non-yrast states





Compare to:

a) Traditional Band Mixing Picture

With no ΔK=0 Mixing

$$B(E2;I_i \to I_f) = \langle I_i 020 | I_f 0 \rangle^2 M_1^2$$

With ΔK=0 Mixing

$$|\tilde{0}_1\rangle \approx |0_1\rangle - \epsilon_0 I(I+1)|0_2\rangle$$

$$|\tilde{0}_2\rangle \approx |0_2\rangle + \epsilon_0 I(I+1)|0_1\rangle$$

and,

$$B(E2;I_i \to I_f) = \langle I_i 020 | I_f 0 \rangle^2$$

$$\times \{M_1 + M_2[I_i(I_i+1) - I_f(I_f+1)]\}^2$$

where,

$$M_2 = \sqrt{\frac{5}{16\pi}} \epsilon_0 e Q_0$$

b) Define New Scales in X(5) Picture

X(5) predictions scaled (÷1.7)

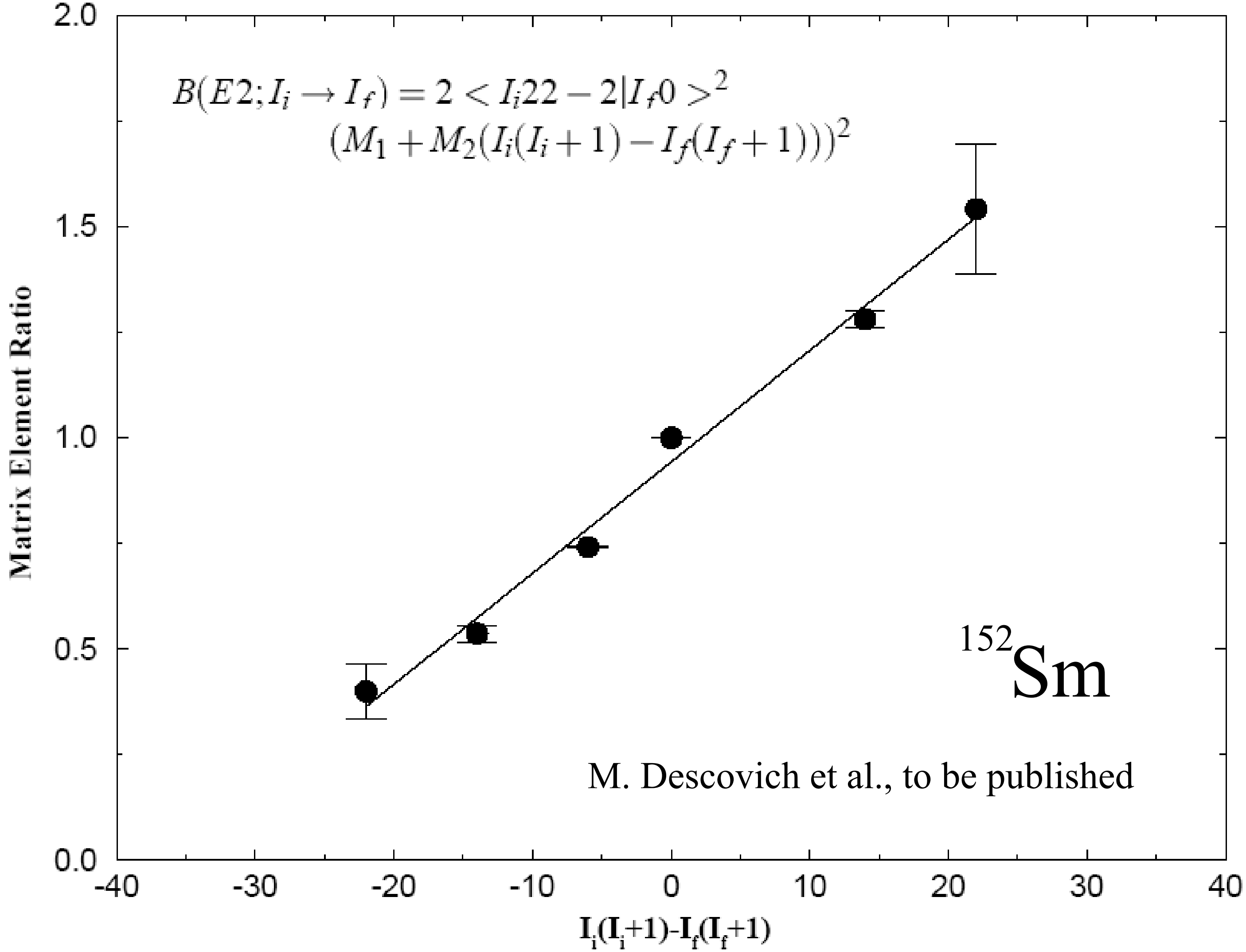
Same scaling factor works in all N=90 isotones

R.M.Clark et al., PRC 67 (2003) 041302

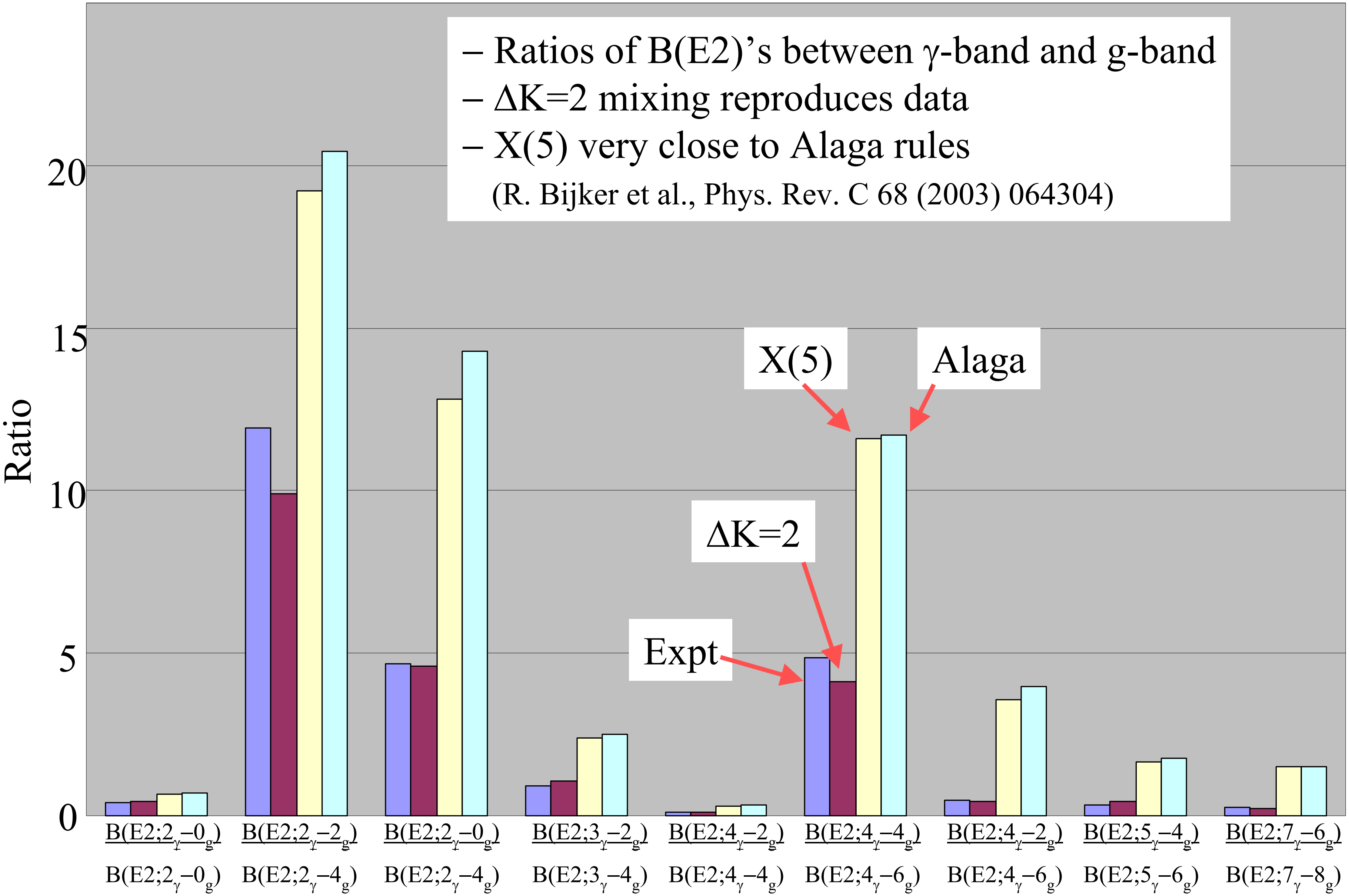
Comment, R.F.Casten et al., PRC 68 (2003) 059801

The γ-Bands in X(5) Candidate Nuclei

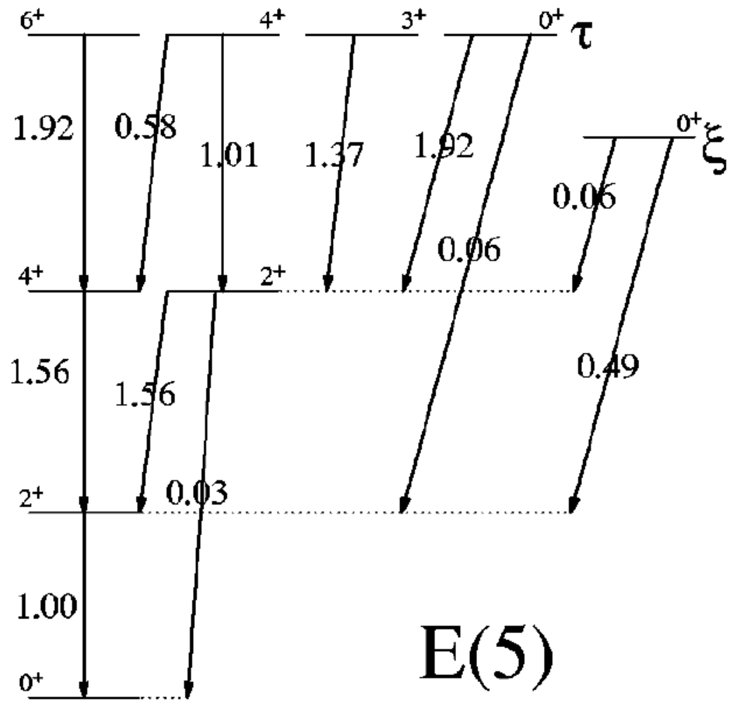
Mikhailov plots for γ-band to g-band transitions, indicate that a $\Delta K=2$ mixing picture reproduces known data in N=90 isotones.



X(5) vs. Band Mixing in the γ Degree of Freedom



E(5): Spherical Vibrator to γ-Soft Rotor Characteristics



a) Energies of the yrast states [$E(4_1^+)/E(2_1^+) \approx 2.20$]

b) B(E2)'s of the yrast transitions

c) Two excited 0^+ states at 3-4 times $E(2_1^+)$

d) E2 decay of 0_τ^+ state predominantly to 2_2^+ level

e) E2 decay of 0_ξ^+ state predominantly to 2_1^+ level

Searching for E(5) Behavior: Spherical Vibrator to γ-Soft Rotor

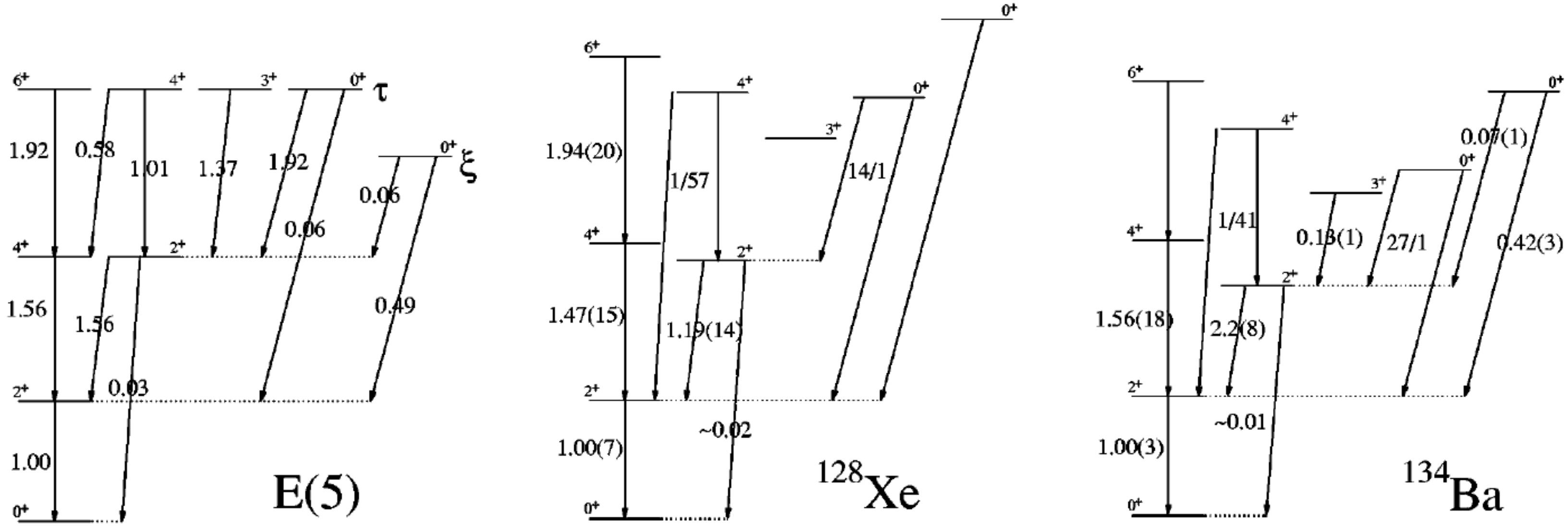
70 Candidates with $2.00 \leq E(4_1^+)/E(2_1^+) \leq 2.40$ (remember the peak)

Only six candidates have their two, lowest, firmly assigned, 0^+ states in range 2.5-4.5 × $E(2_1^+)$: ^{102}Pd, ^{106,108}Cd, ^{124}Te, ^{128}Xe, ^{134}Ba

^{102}Pd: N.V.Zamfir et al., Phys. Rev. C 65 (2002) 044325

^{134}Ba: R.F.Casten and N.V.Zamfir et al., Phys. Rev. Lett. 85 (2000) 3584

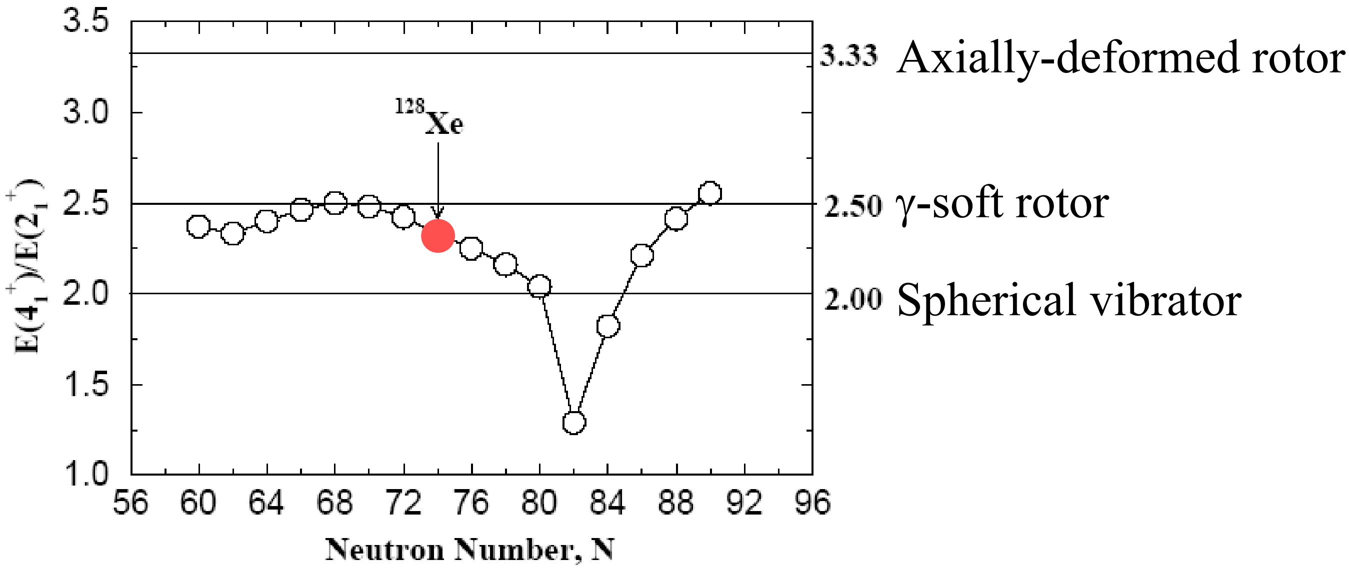
^{128}Xe seems like good new E(5) candidate nucleus



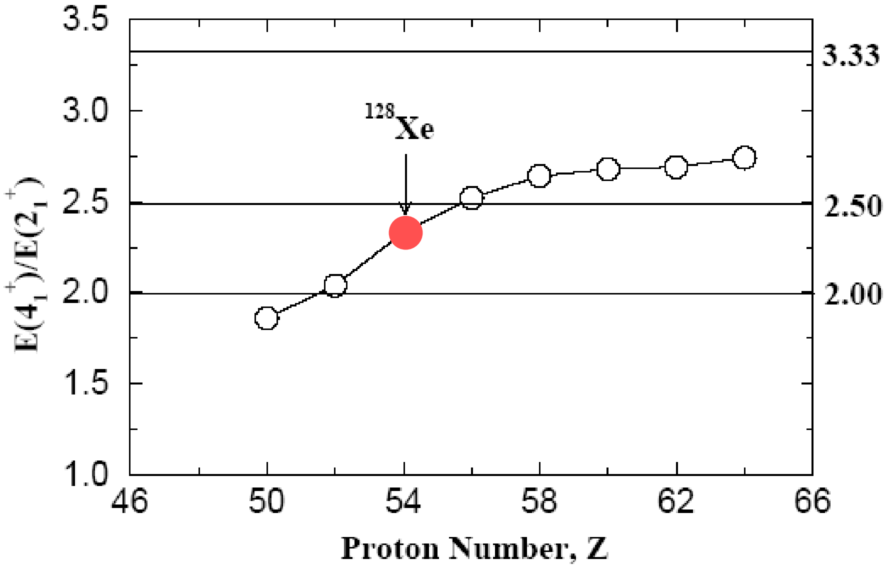
R.M.Clark et al., Phys. Rev. C 69 (2004) 064322

^{128}Xe lies at transition point for both Z and N chains

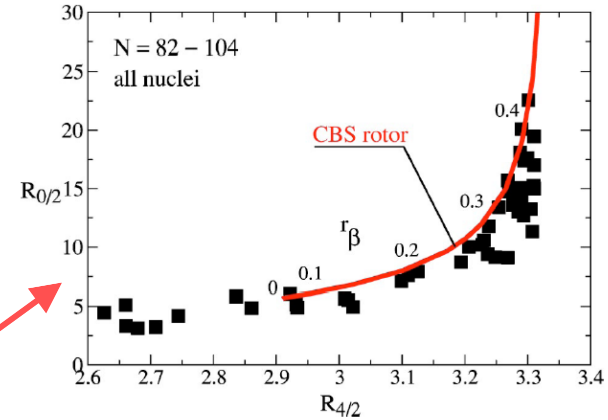
Xe Chain



N=74 Chain



New Directions



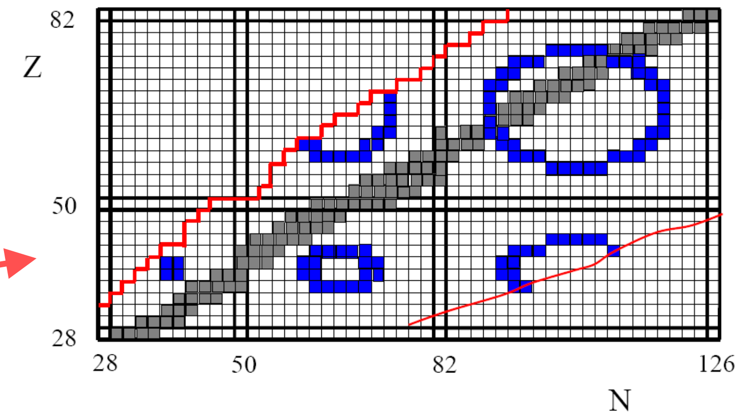
1) Perturbing the Critical-Point Descriptions

Consequences of wall stiffness for a β-soft potential
M.A.Caprio, Phys. Rev. C 69 (2004) 044307
Finite well solution for the E(5) Hamiltonian
M.A.Caprio, Phys. Rev. C 65 (2002) 031304
Evolution of the "β excitation" in axially symmetric transitional nuclei
N.Pietralla and O.M.Gorbachenko, Phys. Rev. C 70 (2004) 011304

2) Exploring New Candidates

Low spin states in 162Yb and the X(5) critical-point symmetry
E.A.McCutchan et al., Phys. Rev. C 69 (2004) 024308

$$P = \frac{N_p N_n}{N_p + N_n} \sim 5$$



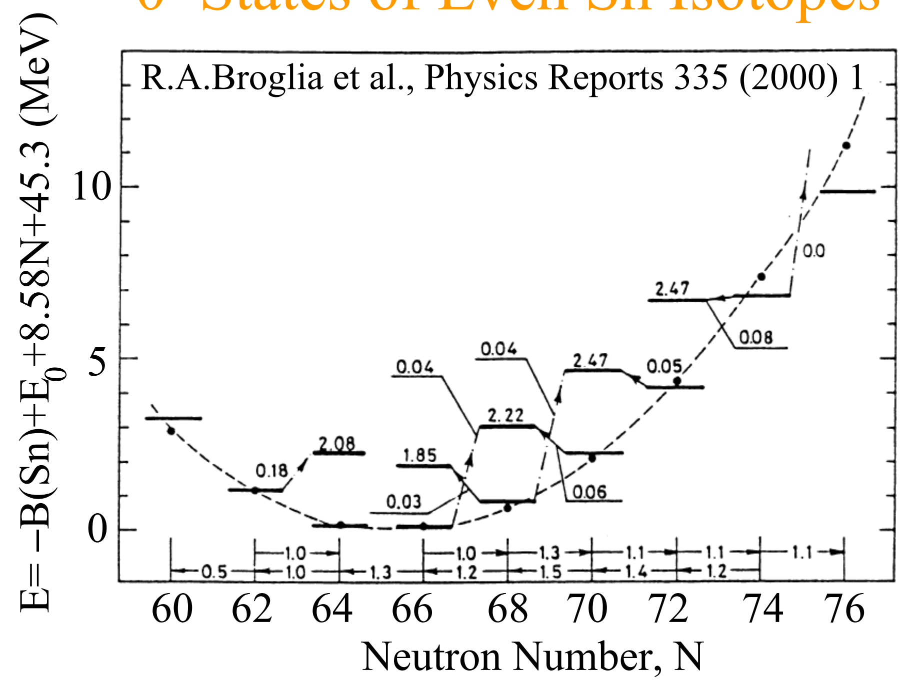
3) Applications to Different Systems/Phenomena

Z(5): Critical point symmetry for the prolate to oblate nuclear shape phase transition
D. Bonatsos et al., Phys. Lett. B 588 (2004) 172
Molecules and atomic clusters?
The Pairing Phase: From Pairing Vibrations to Pairing Rotations?

The Pairing Phase:
From Pairing Vibrations to Pairing Rotations

Shape Transitions	Pairing Transitions
$\mathcal{R}(\theta)=\exp(-iI\theta)$	$\mathcal{G}(\phi)=\exp(-i\mathcal{N}\phi)$
Angular Momentum, I	Particle Number, $\mathcal{N}$
Rotational Frequency, ω	Chemical Potential, λ
$\frac{dE}{dI} \sim \omega$	$\frac{dE}{d\mathcal{N}} \sim \lambda$

0^+ States of Even Sn Isotopes



Solve Collective Pairing Hamiltonian (D.R.Bes et al., NPA 143 (1970) 1)

$$\left\{-\frac{\hbar^2}{2B}\frac{\partial^2}{\partial\alpha^2}-\tfrac{1}{4}\hbar^2\left(\frac{1}{\mathscr{I}B}\frac{\partial\mathscr{I}}{\partial\alpha}-\frac{1}{B^2}\frac{\partial B}{\partial\alpha}\right)\frac{\partial}{\partial\alpha}+\left(V(\alpha)+\frac{\hbar^2M^2}{2\mathscr{I}}-\mathscr{E}\right)\right\}\varphi_{n,M}(\alpha)=0,$$

(α is a deformation of the pair field)

Summary

- Critical-point descriptions involve analytic solutions of Bohr Hamiltonian with only a few free parameters.

On the down side:

- No "perfect" example of critical-point nuclei. (Searches for X(5) and E(5) behavior).
- Traditional descriptions (more free parameters) are often better. (Band-mixing picture).

On the up side:

- Useful benchmarks for perturbative corrections. (For example, the Confined Beta Soft model).
- Insight into tough, old problems. (Nature of low-lying excitations in transitional nuclei).
- Application to other transitional systems and phenomena. (For example, from pairing vibrations to pairing rotations).